Unit II
Growth and Finance

At the conclusion of each class, create a cheat sheet here to summarize the material.

1 Linear Growth and Decay

2 Exponential Growth and Decay

3 Logistic Growth and Decay

4 Interest

5 Annuities and Loans

6 Excel Usage and Variable Rates

7 Applications

Project: Sarah White and her lottery winnings

Scenario: You work for a financial planner who has a client named Sarah White. Sarah has was about to buy a new house for \$155,000, and she was planning to do a 20% down payment (which she had already saved) and get a 30-year mortgage at 3.94% (including fees) paid monthly. Right before she closed, she found out she won the lottery! If she take the lump sum payout, it will be \$124,000 cash (after tax).
Everyone she knows has told her to use the lotto money to buy her house instead of financing it, but she checked with your boss just in case. Your boss believes it may be better for her to put her lottery winnings into a managed investment account, which is anticipated to earn about 6.5% per year (after fees), and then take out the mortgage. Your has assigned you to run the numbers for Sarah and write a short report explaining the results.

Your tasks: Do a little background research. Sarah has heard the term "fiduciary responsibility" and wants to know what that means. In general, what is a financial planner? What will your boss do for Sarah?
Explain how managed investment accounts work and how mortgages work.
Do a little mathematical analysis. Sarah's options are:

- Option 1: Forget the mortgage, just buy the house! In 30 years, she has . . . a house.
- Option 2: Finance the house and put the lotto money in an investment account. Deduct the monthly mortgage payments out of this account. In 30 years, she has . . . a house, and an account with (maybe) some money in it.

Product: Construct a well-written letter to Ms. White (about 1 page, double-spaced) with the usual sections - an introduction with an explanation about what a financial planner is and can do for her, an explanation of what mortgages and investment accounts are and how they work, a summary of your calculations with a recommendation, and a couple of concluding sentences. Make it look professional ? imagine what you would actually hand to a client (ex, use a decent letterhead/template, use professional language, etc.).

Submission Guidelines: Submit online under the appropriate assignment. Use good grammar and writing.
Include citations for your definitions and giving Sarah a place to look up more information. You wouldn't do it in real life, but include a separate sheet where you show your work for calculating Sarah's mortgage balance and investment account balance. You may assume her mortgage interest is compounded monthly.

Day 1 - Linear Growth and Decay

Marco's Bottle Collection

Marco collects antique soda bottles. His collection currently contains 437 bottles. Every year, he budgets enough money to buy 32 new bottles.

1. Translate this from words into math notation:

$$\text{starting value} = \underline{\hspace{2cm}}$$
$$\text{next month} = \text{this month} + \underline{\hspace{2cm}}$$

2. Estimate the number of bottles he will have each year:

Year	Bottles
0	
1	
2	
3	
15	
n	

3. We'll make the equation from Question 1 more formal. Fill in the blanks to create a formula for B_n:

$$B_0 = \underline{\hspace{2cm}}$$
$$B_n = B_{n-1} + \underline{\hspace{2cm}}$$

   This is called a **recursive equation** or **model** for B_n.

4. Fill in the blanks to create a formula for B_n, the number of bottles in Marco's collection in year n:

$$B_n = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} * n$$

   This is called an **explicit equation** for B_n.

5. How many years will it take for his collection to reach at least 1000 bottles?

6. You run into Marco in year 3. Just as he expected, he has been steadily buying about 32 new bottles a year and plans to continue doing so. He tells you he actually has 535 bottles in his collection. Is there a problem?

7. How many bottles does this equation predict that Marco will have in year 100? Does this number have any meaning?

Marco's bottle collection is modeled as **linear growth**: it grows by (about) a constant amount each year. The name comes because, if we graph it, it looks like a line.

Isabella's Coin Collection

Marco's friend Isabella purchased a large collection of coins at an estate sale and is selling them individually in her online store. The number of coins she has in inventory decays linearly according to the model (C is the number of coins, n the number of days since she started selling):

$$C_n = 212 - 59n$$

8. How many coins were in the original collection?

9. How many coins does Isabella sell each day?

10. Graph this equation by hand and then check it with a computer graphing program or calculator. *(Hint: It may help to write it as* $y = -59x + 212$*.)*

11. What is the **slope**? Describe it in your own words and identify it in the equation.

12. What is the y**-intercept**? Describe it in your own words and identify it in the equation.

13. The input variable is called the **explanatory** or **independent variable**, and the output variable is called the **response** or **dependent variable**. Define these in your own words and identify them in the equation.

Stephen's Garden

Marco and Isabella have a third friend, Stephen, who is an avid gardener. He grows pumpkins and is harvesting them to give away for carving near Halloween. He had 19 pumpkins on Oct 1 and only 4 remaining on Oct 31.

14. On average, how many pumpkins does he harvest daily? Does this number make sense?

15. Give a model (an explicit equation) for P_n, the number of pumpkins remaining in Stephen's garden on day n in October.

Stay-at-Home Fathers

The number of stay-at-home fathers in Canada has been growing steadily. While the trend is not perfectly linear, it is fairly close.

1976	19084	1991	2000	2010
20,610	29,725	43,530	47,665	53,555

16. Find an explicit formula for the number of stay-at-home fathers. (Note: there are several different reasonable ways to do this.)

17. Estimate the number of stay-at-home fathers in 2005.

Using a model to estimate a new point between known data points is called **interpolation**.

18. Predict the number of stay-at-home fathers in 2020.

Using a model to predict a new point outside the range of known data is called **extrapolation**.

19. Use the model to predict the number of stay-at-home fathers in 2050. Are you more confident about this number or about the 2020 prediction? Why?

20. Did the other students do part (16) the same way you did? Did they get similar answers? Is there a problem?

Note: If something is perfectly linear, it doesn't matter which points you pick to calculate the line; if it's not, then how good the line is depends on how linear the data is. We'll discuss this point again in later units when we talk about regression.

Answers: 2. 437, 469, 533, 917. 5. 18 years (note the rounding). 7. 3,637. 8. 212 coins. 9. 59 coins sold per day. 11. -59. 2e. 212. 13. x (or n) is explanatory, y (or C_n) is response. 14. 0.5 pumpkins.

Day 2 - Exponential Growth and Decay

Bacterial Growth

You are growing some bacteria in a petri dish in biology lab. You record their populations hourly.

Hour	0	1	2	3	4
Population	138	159	182	210	241

1. Graph the data points.

2. Assume this colony demonstrates linear growth. Draw a line that you think matches the data as well as possible. Does the model look like it fits the data well? Why or why not?

3. Find a linear model (recall: an explicit equation) for the population.

4. Predict the population at hour 5 using your model. At hour 5, you actually count 278 bacteria. How far off were you?

Some values tend to grow based on how large they are rather than by a constant value each hour. One example of this is **exponential growth**, where a value grows by a constant percentage rather than a constant number, as with your bacteria. Say your population starts at 138 bacteria and grows by 15% each hour.

5. Calculate the size at each hour:

Hour	0	1	2	3	...	15	...	n
Population								

6. Do these numbers match the numbers in the original table better?

7. At hour t, you have population P_t, and the colony grows at 15% each hour. How much will it grow in the next hour?

8. Give a recursive formula for the population (tell what P_0 is and give a formula to find P_t from P_{t-1}):

9. Give an explicit formula or model for the population (give a formula for finding P_t directly from t):

Population Growth

Human populations often grow exponentially as well. India is the second most populous country in the world, with a population in 2008 of about 1.14 billion people. The population was growing at that time at about 1.34% per year.

10. Find an exponential model for India's population (let P_n be in billions of people and n years since 2008).

11. If this trend continued, what would India's population be in 2020?

12. In reality, India's population in 2020 was about 1.339 billion. Is this number the same? What could cause the difference?

Financial Growth

A friend has decided that the semester tuition at a local college grows exponentially, too, and has modeled it with the equation

$$T_n = 4600(1.072)^n$$

where time n is measured in years since 2010.

13. What was tuition in 2010?

14. How fast (in %) does tuition grow each year under this model?

Solving Equations with Exponentials

You may need to solve some equations involving variables in the exponent. You could solve these using logs, or you could use an equation solver (there are many available online). Find one and solve these.

15. $8 = 2^t$

16. $200 = 5 \cdot 1.03^t$

17. $17102 = 1000 \cdot 1.0212^t$

18. $17 = t^2$

Social Networking

The number of members of a social networking site was 45,000 in February 2010 when they officially went public and is expected to grow exponentially at a rate of r% per month.

19. Give a formula for U_n, the number of users n months after February 2010.

20. The site has 60,000 users in October 2010. Find r and rewrite the formula appropriately.

21. When will the site reach 100,000 users?

Time to Double

You start a new experiment with the same bacteria you used previously; they are still growing at 15%, but, this time, you start with 300.

1. Model the new bacterial population. How long does it take these bacteria to double in size?

2. How long did the bacterial population in Question 9 take to double? How do these answers compare?

3. If your friend's bacteria are growing at a rate of 20% per hour, how long will they take to double? Does it matter what size your friend's bacterial colony is?

Answers: 11. 1.337 billion people. 13. \$4600. 14. 7.2%. 15. 2. 16. 124.80. 17. 11.95. 18. 4.12. 20. r=0.037. 20. $n = 22$, or Dec 2011. 1. 5.0 hours. 2. 5.0 hours. 3. 3.8 hours.

Day 3 - Logistic Growth and Decay

Note: A slightly different presentation of logistic growth is contained in your book on p. 188-191, but it focuses on populations which grow discretely (such as plants which reproduce once annually), whereas the model below is for populations which grow continuously.

The Flu

One person in a small town of population 1,000 catches a particularly nasty strain of the flu. Several days in, doctors realize the population of sick people is increasing quickly at a rate of about 12.5% per day.

1. Give an exponential growth model for the population of sick people.

2. Estimate the number of sick people at 20 days.

3. Estimate the number of sick people at 60 days. Is there a problem?

4. A weakness of the exponential growth model is that it always predicts that populations will continue increasing without bound. In reality, any stable environment has a carrying capacity, that is, a maximal population that can be supported in that environment. The carrying capacity for a given system is governed by things like physical space or the food and water supply. Does that apply here?

A growth model that accounts better for the carrying capacity is the **logistic growth model**, in which growth occurs exponentially at first and then slows as it approaches the carrying capacity. The logistic growth model is:

$$P_t = \frac{K}{1 + ae^{-bt}}$$

where K is the carrying capacity; a is a number chosen so that the initial value is correct, that is, $a = K/P_0 - 1$; and b is the logistic growth rate (which is usually given to you). Note that e is actually a number, not a decimal (it's about 2.7; you should have a button for it on your calculator). For the population of people with the flu:

5. Find the carrying capacity K, the initial population P_0, and the number a.

6. If scientists estimate the logistic growth rate of this flu is $b = 0.6030$, write down a logistic model for the sick population.

7. Use this model to estimate the number of sick people at 0 days, 20 days, and 60 days. Do these make sense?

Exponential vs. Logistic Growth

Compare your exponential and logistic models.

8. Use a graphing calculator or computer program to graph the logistic model and the exponential model. Draw them here.

9. Which model seems more appropriate for modeling the spread of flue in this town, the exponential or the logistic model? Why? Are there other situations where a different model may be more appropriate to model the spread of a disease?

Modeling World Population

You are part of a group compiling a report on world progress towards sustainability goals, and you have been tasked with collecting data on world population and summarizing it for the group to use later when writing the report. You have found a study by the UN (New York Times, November 17, 1995) showing the year in which world population passed or is projected to pass each billion people from 2 billion to 10 billion.

1927	1960	1974	1987	1999	2011	2025	2041	2071
2	3	4	5	6	7	8	9	10

10. Construct a scatterplot of the data, using the input (horizontal axis) variable t, years since 1900, and the output (vertical axis) variable P, world population in billions of people.

11. What type of population model is appropriate for this data? Does it look right for your data? Does it make sense with what you know about growth types?

12. Which constants will you need to know to produce a model? Can you estimate them based on the data you have?

13. Produce a model.

14. Estimate the world population in 1900 and the current year based on your model and compare them to the actual numbers. Estimate when the world population will reach 15 billion.

15. Are these examples of interpolation or extrapolation? How accurate were your estimates?

Answers: 2. 11. 3. 1,173. 5. $K = 1000$, $P_0 = 1$, $a = 999$. 7. 1, 994, 1000.

Day 4 - Interest

Compound Interest

You put \$1000 in the bank earning 6% interest each year.

1. (a) Find an explicit model for your balance B_t.

   (b) When do you have \$1500?

\$1000 is called the **principal**, the amount your account starts with. When interest is calculated once per year like this, we say it is **compounded annually**, and 6% is the **annual interest rate** or **annual percentage rate (APR)**.
You find a new bank that will calculate your interest every month: that is, it will give you your 6%, but it will divide it among 12 months. Then 6% is still called your **APR**, and each month you will get $6\%/12 = 0.5\%$ interest.

2. Find an explicit model for your balance B_m where m is measured in months.

3. We usually give models for financial situations in terms of years, not months. Find an equation for B_t where t is measured in years.

4. How much interest do you earn in the first year? How does this compare to the previous bank?

5. When does your account hit \$1500? How does this compare to the previous bank?

Continuously Compounded Interest

You notice that compounding more often means you get more money. If you start with \$1000 and earn 6% interest, in one year you'll have \$1060; if it compounds monthly, you'll have \$1061.67; weekly, \$1061.80; daily, \$1061.83. The improvement seems to be slowing down, and your balance seems to be approaching some magical max.

6. Guess what your balance will be if you earn interest every nanosecond.

You find a bank that claims they'll figure out that magical max, where they compound your money more and more frequently. This is called **interest compounded continuously**, and it follows the equation:

$$B_t = B_0 e^{rt}$$

7. Model the balance from the previous problem if it is compounded continuously.

8. What will your balance be in 1 year?

9. When will your account balance have doubled?

10. If your friend deposits \$11,245 into an identical account (earning 6% interest compounded continuously), how long will it take that account to double?

Some Home Improvements

11. You estimate you will need \$10,000 for a new roof in 2 years. If you receive a \$7500 bonus this year and deposit it into an account earns 4% annually, will it be enough? If not, how much do you need to deposit?

12. You purchase a washing machine for \$395, and the department store offers to finance it for you at 4% interest, compounded monthly, with no payments required for the first year. If you take this loan, how much will you owe at the end of the first year?

13. You are saving to buy your first house, and you estimate you will need a down payment of \$35,000. You inherit \$29,000 and put into an account averaging 5.7% compounded continuously. When will your account be large enough to cover the down payment?

14. You would like to make a minor car repair which will cost \$500, and you are considering whether to wait and save the money or take out a loan. Your mechanic has a partnership with a loan company which will loan you the \$500 for two months for \$30 interest. What percentage interest did they charge for these two month? What is the annual interest rate (APR) for this loan? Will you take it?

Answers: 1b. about 6.95, so at year 7. 4. \$1061.68. 5. about 6.77 years, i.e., 6 yrs 10 mos. 8. \$1061.84. 9. 11.6 years. 10. 11.6 years. 11. no; need at least \$9,245.56. 12. \$411.09. 13. 3.3 years. 14. 6%, 36% .

Day 5 - Annuities and Loans

Annuities

You graduate from college and open a retirement account. You decide to deposit \$50 each month. You estimate that you will get an interest rate of approximately 3% compounded monthly (that is, with an APR of 3%).

1. Calculate the value of your account for each of these months.

Month	Starting balance	After interest	After withdrawal
0			
1			
2			
3			

2. Find a recursive model for your balance B_t.

This is called an **annuity**, where you earn interest and also make regular deposits. Most retirement plans like 401ks and IRAs work this way (although most will get better APRs when the account has a little more money!) If you open an account with \$0, earn interest r% (written as a decimal) compounded k times per year, and make deposits of size \$$d$ (also k times per year), then the future value of your annuity after t years is given by:

$$B_t = \frac{d\left((1+\frac{r}{k})^{kt} - 1\right)}{\frac{r}{k}}$$

3. Find an explicit model for the balance B_t of your account.

4. How much money will you have in 10 years?

5. Your neighbor Bob also saves $50 each month, but he doesn't trust banks so he puts it under his mattress. How much money does he have in 20 years? How does this compare to your account? (Note: If Bob trusted banks and put his money is a savings account, there's a good chance he'd earn exactly the same interest rate that his mattress is paying him.)

6. You're 45 and hope to retire in 20 years. You have some savings already, but you anticipate a shortfall of approximately $200,000. You open a new retirement account (projected to earn 5% APR) and plan to set up direct deposit biweekly from your paycheck. How much money do you have to deposit each month into this account?

Payout annuities

Sometimes we consider the opposite situation, where you start with a principal which earns interest, and then you slowly withdraw money over the years. You have now retired with your $200,000, still earning 5% interest compounded monthly, but now you're withdrawing $3,000 a month to cover your expenses.

7. Calculate the value of your account for each of these years (note: assume the bank calculates interest before you make the deposit). Let Year 0 be the year you open the account (in particular, you start without any money in it).

Month	Starting balance	After interest	After withdrawal
0			
1			
2			
3			

8. Find a recursive model for your balance B_t.

This is called a **payout annuity**. Many insurance payouts and lottery payments work this way. There are several different ways to calculate the different values of a payout annuity. One special case is a formula we use if we want a payout annuity to last a particular amount of time. There is an explicit formula for this kind of annuity that relates the initial deposit B_0, the APR $r\%$, the withdrawal amount $\$d$, compounding k times a year, and the number of years it lasts t:

$$B_0 = \frac{d\left(1-(1+\frac{r}{k})^{-kt}\right)}{\frac{r}{k}}$$

9. How long will your account last?

A **loan** is a special type of payout annuity where you owe money and make regular payments to reduce your loan amount but are also charged interest which increases your loan amount.

10. You can afford \$200 a month as a car payment. If you can get an auto loan at 3% interest for 60 months (5 years), what size loan can you pay afford to get?

Example

An **endowment** is a gift to a nonprofit which is invested and the interest is used for some charitable purpose.

11. A nonprofit receives a gift of $75,000 decides to start an endowment to hire an intern every summer. If they anticipate earning 5% compounded annually and withdraw just enough that the account balance stays the same, how much do they pay the intern?

12. The nonprofit realizes they need to be paying the interns more. They decide to withdraw enough from the endowment that it will be used up in 20 years. Now how much will they pay the intern?

13. Another donor offers to donate $5,321.43 to the nonprofit every year, which they will invest without touching for 25 years. How much will they have after that time?

14. What is the difference between Questions 11, 12, and 13? What was similar or different about the formula and the constants?

Answers: 1. $0, $0, $50; $50, $50.125, $100.125; $100.125, $100.38, $150.38; $150.38, $150.75, $200.75. 4. $6,987.07. 5. $6,000. 6. $224.18. 7. $200,000, $200,500, $197,500; $197,500, $197,993.75, $194,993.75; $194,993.75, $1954,451.99, $192,451.99. 9. 6.5 years. $7358.18. 10. $11,130.47. 11. $3,750. 12. $5,321.43. 13. $253,967.

Day 6 - Excel Usage and Variable Rates

Compound Interest

You put $1000 into an account earning 3% interest each year. We will set up a spreadsheet to calculate the value of the account each year.

1. Open a new spreadsheet in excel.
2. We'll need a column for time. We'll use the column named A.
   (a) Type "time (years)" into the first cell (cell A1).
   (b) Set up this column to display the years 0 and 1. There are many ways you can do this besides just typing in the numbers. Try each of them:
      - type the numbers in
      - type 0 into the first cell (which is named A2), then go into the next cell (A3) and type the equation "=A2+1"
      - type 0 into the first cell (A2), then go into the next cell (A3) and type "=", then click on the cell you want (A2), then type "+1".

   (c) Now set it up to display the years 0 to 10. Highlight the cell with the formula, hover the mouse over the bottom right-hand corner of the highlighted cells until it changes into a little cross, then click, drag your mouse down, and release.
3. We'll need another column (use column B) for balance.
   (a) Type "Balance" into the first cell (B1).
   (b) Enter the initial balance of $1000 for year 0 in box B2.
   (c) In year 1, enter an appropriate formula (ex, "=B2*1.03"). Drag down the formula to fill in the column up to year 10.
   (d) Reformat your balance column as currency (note: it's easiest to do it by reformatting the whole column rather than just highlighting the cells you need; click on the column name and then click on the dollar sign in the top menu).
4. Using your spreadsheet, what is your balance in 10 years?

5. Write down an explicit formula to calculate your balance in 10 years. Verify that the answer matches.

What if you put $1000 into an account earning 3% interest compounded monthly, not annually?

6. Copy the work you've already done.
   (a) Highlight the two columns you used before (note: just click on the column headings to get the whole columns instead of trying to highlight exactly the right cells).
   (b) Copy and paste them into some new columns.
   (c) Insert a new row above your work (right-click on the title of row 1, then click "insert").
   (d) Use the new row to title these problems "Annually" and "Monthly."
7. Alter your spreadsheet to change years to months.
   (a) Change the name.
   (b) You may need to resize the column if the name doesn't fit: move your mouse over the title of the column and hover over the right-hand edge of the cell. Your pointer will change from a black down arrow to a thin black line with double sideways arrows. Drag it right/left to resize. Double click to automatically resize.
   (c) Drag down the month column until it lasts 10 years.
   (d) Correct the formula in your balance column to account for months instead of years.
8. Using your spreadsheet, what is your balance in 10 years?

9. Write down the formula to calculate your balance in 10 years and verify that the answer matches.

Annuities

You start an investment account (earning 6% interest compounded monthly) with $1000. You will also be depositing $500 each month. We'll model this account until it hits $100,000.

10. Make columns for "time (months)," "starting balance," "balance after interest," and "balance after deposit."

11. Fill in the time column as above.

12. Fill in ending balance for month 0 and leave the other cells in that row blank.

13. Fill in the row for month 1

    (a) Starting balance (do NOT type it in; reference the appropriate other cell).

    (b) Balance after interest (do NOT calculate a decimal on a calculator: make excel do the work with an equation).

    (c) Balance after deposit (again, try to use a formula, not type in a number).

14. Highlight all the cells in month 1 and drag them down.

15. Extend your spreadsheet until you break $100,000. How long does it take?

Now real life intervenes and affects these numbers a little. Since we're using a spreadsheet and not a single formula, we can take that into account here very easily.

16. Start a new problem: copy the columns.

17. In month 11, your car breaks, and you can't make the usual 500 deposit; in fact, you have to withdraw $312 to cover the repair bill. Alter your spreadsheet.

18. In month 13, you get a bonus and deposit an extra $1700. Alter your spreadsheet.

19. After 4 years, your investment advisor informs you that you now have enough money to get into a slightly nicer category of account; starting in month 49, your interest rate increases to 6.5% (still compounded monthly). Alter your spreadsheet.

20. Extend your spreadsheet until you break $100,000. How long does it take?

Payout Annuities and Loans

You take out a car loan for $10,000.

21. You get a special deal: it accrues interest (the principle increases) at a rate of 4% compounded monthly, but you do not need to make any payments for five years. Make a spreadsheet to show the monthly balance for those first five years. Make a new spreadsheet. How much do you owe in 5 years?

22. A different company offers a similar deal, financing at 4% compounded monthly, but you would have to make a payment of $167/month (note: if you paid $10,000 evenly over 60 months, it would be about $167/month). Make a new spreadsheet. How much do you owe in five years?

23. You talk to a third company that again offers a deal at 4% compounded monthly, and their offer is really special: they give you a monthly payment so that your balance after 5 years is almost exactly $0.00.

    (a) Make a guess about what your monthly payment might be.

    (b) Fill in your guess in the spreadsheet. Is it right? If not, adjust it. (Note: You'd probably like to be able to see the last cell without scrolling down. Hide some rows in the table - highlight them by clicking on its row names, then right-click and select "hide.")

    (c) What is the monthly payment?

Note: This is what they actually do in real life for car loans, student loans, mortgages, etc! It is called an **amortized loan**.

(d) If you round up your payment to the next hundred dollars, how much faster do you pay off the loan?

(e) If you round up your payment to the next hundred dollars, how much money do you pay over the life of the loan? How does this compare to Question 23d?

24. You have \$5,700 in credit card debt with an average APR of 12.51% compounded monthly. A standard credit card will require a minimum monthly payment of the (monthly) interest + 1% of current balance OR of \$5.00 (whichever is bigger). How long does it take to pay it off if you just make the minimum payments?

Note: You can either change the monthly payment by scrolling down and changing the formula manually, or you can use the "minimum" function in excel; check the help option if you haven't used it before.
Note: This is the average credit card debt for a US household and a standard APR.

Answers: 4. \$1343.92. 8. \$1349.35. 15. in month 137 (11.4 yrs). 20. 133 months (11.1 yrs). 21. \$12,209.97. 22. \$1,138.04. 20c. \$184.17 (note: \$184.16 leaves you just a bit short; the loan company would rather have you overpay and then just bill you less in the last month) 23d. in 55 months. 23e. \$10,957.57 (\$92.31 less). 24. 394 months (32.8 yrs).

Day 7 - Applications

Try to do these problems both using the equations and using excel.

1. You plan to retire at 60. After retiring, you want to be able to take out $1500 every month from your retirement account; you would like the account to last at least until you are 90.

   (a) The account earns 5.5% interest. How much will you need in your account when you retire?

   (b) If you start this savings account when you are 30, how much will you need to deposit every month?

2. You buy a house for $187,500 (note: this was approximately the median home price in Omaha/Council Bluffs in 2019). You put 20% down and get a mortgage at 3.1% (note: this means you provide 20% of the cost from your savings, and the bank loans you the rest with an APR of 3.1%; since you're making monthly payments, we'll say it's compounded monthly).

   (a) What is the initial balance on your mortgage? (Note: in real life, you'd also have several thousand dollars of closing fees thrown onto the initial balance.)

   (b) What is the monthly payment on your mortgage?

(c) How much is your remaining balance after 10 years? *(Hint: at that point, you essentially have a 20 year loan left at 3.1% interest with the monthly payments in part* (2b).*)*

(d) How much did you pay in total over the life of the loan?

(e) The money you paid went partly to repay the principal (the original amount you borrowed) and partly to cover interest. How much did you pay in total towards the principal? How much did you pay in total towards interest? What percentage of your total payments went towards interest?

3. You have a 30-year mortgage for $100,000 at 3% interest.

   (a) What is your monthly payment?

(b) Your first month, how much interest did your mortgage accrue? What percentage of your first monthly payment went to cover that interest?

(c) What was your balance at the beginning of the very last month?

(d) How much interest did your mortgage accrue in that last month? What percentage of your last payment went towards interest?

4. You put a $1200 emergency car repair on your credit card at 18.61% APR compounded monthly (note: this was the average APR on a credit card in first quarter of 2020). You make a monthly payment of $30. How long will it take you to pay it off?

Answers: 1. $264,182.64; $289.16. 2a. $150,000. 2b. $640.52. 2c. $114,456.87 2d. $230,587.20. 2e. $150,000; $80,587.20; 35%. 3a. $421.60. 3b. $250; 59% 3c. $420.55. 3d. $1.05; 0.25%. 4. 5.24 years.