

Unit III

Voting

At the conclusion of each class, create a cheat sheet here to summarize the material.

1 Plurality and Elimination Systems

2 Scoring and Pairwise Comparison Systems

3 Approval Voting

4 Fairness

5 Unfairness

Project: Electoral Systems

Scenario: You work for a specialty consulting firm specializing in the voting process. Your firm currently has three projects underway, and you need to help them evaluate one of them using the four Fairness Criteria and decide which voting system would be the best method to reflect the will of the voters. The scenarios are:

- A company is hiring a new manager. It has 20 serious candidates and needs to whittle them down to 3 finalists for on-site interviews.
- A company is selecting the location of a new HQ. It needs to pick from 5 candidate cities.
- A state is rewriting its election laws for Governor during a gubernatorial election (weird, right?). There are 5 candidates, and the people of the state need to elect one to be Governor.

Your tasks: Select a scenario. Think about how to ensure that the outcome of the election best represents the opinions of the voters; in particular, consider the four fairness criteria and why they would (or wouldn't) be important in this situation. For example, for the majority criterion, what would it mean if someone were ranked first by the majority of the voters? Would it be a problem if this person didn't win the election? What is it that is special about the scenario you're discussing which makes you answer this way? Repeat this question to yourself for each of the four criteria. Then consider any additional concerns you have about fairness, practicality, etc. Choose a voting method.

Product: Create a report (1 page, double-spaced) for your consulting firm to advise which election system should be used for this scenario. It should be appropriate for the intended audience (ex, your audience may not have heard of fairness criteria before). Support your choice of voting system with solid arguments (ex, if you decide that it would be bad if a majority winning candidate lost, then tell me why that matters so much).

Submission Guidelines:

- Submit online under the appropriate assignment. Use good grammar and writing.
- It should include the usual sections - a reasonable title/author list, a brief introduction, a description of the selected election system, an explanation of your reasoning, and a brief conclusion.
- Include citations if you include any information we have not covered in class.
- Make sure you discuss what **could** happen. Do not pick a preference schedule and then tell me what would happen then; instead, consider what kind of preference schedules you might end up with and ask yourself what you would want to happen in each situation.

Day 1 - Plurality and Elimination Systems

We're going to look at several scenarios, examine how a group of voters feels about some candidates, and then try to decide what outcome would reflect the voters' opinions most accurately.

Dinner choice and preference schedules

A vacation club is meeting to pick a destination to visit this year. Before they get to that, though, they decide to order dinner, and they must choose between pizza (P), tacos (T), and sandwiches (S). From chatting beforehand, it becomes clear what everyone wants:

	Ann	Bob	Marv	Alice	Eve	Omar	Lupe	Dave	Jim
1st choice	P	S	S	P	S	P	P	S	P
2nd choice	S	P	P	S	T	S	T	P	T
3rd choice	T	T	T	T	P	T	S	T	S

It would be awkward to write down all everyone's preferences for larger groups of people, so these individual preferences are typically condensed into a table called a **preference schedule** like this:

# people	3	
1st choice	P	P
2nd choice	S	T
3rd choice	T	S

- Let's fill in the preference schedule.
 - There are 6 possible ways a voter could order P, S, and T, but only two are filled in. Fill in the rest, and mark how many people prefer each ranking.
 - Add up the number of people listed in the top row of the preference schedule. Does it match the number of people at the meeting?

- Let's pick a winner. Look at the preference schedule. What do you think they should order? Why?

Vacation destination

The vacation club has eaten their dinner and is now trying to decide which destination to visit this year: Honolulu (H), Orlando (O), or Anaheim (A). Here is their preference schedule:

	1	3	3	2
1st choice	A	A	O	H
2nd choice	O	H	H	A
3rd choice	H	O	A	O

3. Let's pick a winner. Look at the preference schedule.
 - (a) Who do you think should win?

 - (b) Is this situation easier or harder to decide than what to order to eat?

4. Ann doesn't see the preference schedule but asks everyone what their favorite choice is. Then she argues that Anaheim should clearly win because the largest number of people say it's their favorite.
 - (a) Is she right that Anaheim has the largest number of people who say it's their favorite?

 - (b) What do you think of this suggestion?

5. Lupe doesn't see the preference schedule but overhears Ann asking everyone what their first choice is. He disagrees with Ann and thinks no city is a clear winner, so he proposes they eliminate Honolulu since it has the fewest first place choices, and then they can reconsider between Anaheim and Orlando in isolation.

(a) How would everyone compare Anaheim and Orlando?

(b) What do you think of this suggestion?

6. Bob doesn't see the preference schedule but asks everyone what their last choice is. Then he argues that Orlando should clearly not win because it's the last choice, so they should eliminate Orlando and have a runoff to decide the winner.

(a) How would everyone compare Anaheim and Honolulu?

(b) What do you think of this suggestion?

7. Jim says he's bankrolling the whole club, so he should get to decide, and he wants to go to Honolulu. What do you think of his argument?

8. Marv figures out what the results will be before the election. He says, hey, there were three of us who preferred Orlando because it's the best (and Honolulu is next, then Anaheim in last). Orlando's obviously not going to win, though, and we really prefer Honolulu to Anaheim, so let's all just fib and say Honolulu is our favorite when we cast our ballots, and then Honolulu will clearly win.

(a) Draw an updated preference schedule.

1st choice
2nd choice
3rd choice

(b) Is he right that this would work?

(c) What do you think of his suggestion?

Definitions

9. Anna's proposal is called **plurality voting**: everyone voter casts one vote, for their top choice. The candidate with the most votes wins. This is one of the most common systems in use.
- (a) Can you think of places this is used in real life?
- (b) Can you think of some pros and cons?
10. Lupe's proposal is called **instant runoff voting (IRV)** or **plurality with elimination** and is a form of **ranked-choice voting** (meaning the voters have to rank all their choices).
- (a) Do you know of places this is used in real life?
- (b) Can you think of some pros and cons?
11. Marv's trick to manipulate the result is called **insincere voting**: some voters change their vote (they don't quite follow their real opinions) to get an outcome they like better.
- (a) Can you think of some modern political situations where people engage in insincere voting?
- (b) Could anyone else in the vacation club use insincere voting to change the outcome?

Answers: 1a. $3P > S > T, 3S > P > T, 2P > T > S, 1S > T > P$ 4a. yes
 5a. $A > O: 6 > 3, A$ wins 6a. $A > H: 4 > 5, H$ wins 8a. $1A > O > H, 3A > H > O, 3H > A > O, 2H > A > O$ 8b. depends ? H is definitely a really strong candidate, though!

Day 2 - Scoring and Pairwise Comparison Systems

Vacation destination, revisited

Recall the vacation club considering whether to visit Honolulu (H), Orlando (O), or Anaheim (A).

1. Alice hears the others' arguments about who should win. She thinks the way people rank candidates matters, and they really do need to pay attention to not just first choices; however, she's getting tired of all the arguments about exactly how to determine the winner, so she suggests they just assign points based on everybody's preferences.
 - (a) Do this: for every time someone is ranked last, give them 1 point; second-to-last, 2 points; and first, 3 points.
 - (b) Who has the most points? Determine the winner.
 - (c) What do you think of this system?

2. Nick notices something interesting and argues that Honolulu really should win: it's clearly preferred to Anaheim (he asks everyone to do a show of hands to see whether they prefer Anaheim to Honolulu), and it's also preferred to Orlando (he has them do a show of hands again for Honolulu vs. Orlando).

(a) Is he right that most people prefer Honolulu to Anaheim? Honolulu to Orlando?

(b) What do you think of his argument?

(c) We formalize Nick's idea with a **Condorcet tournament**, where you compare each pair of candidates head-to-head, make everyone choose between them, and record the result. You can draw a tournament as a graph. Put a dot for each candidate. For each pair, figure out who would win between just the two of them, and draw an arrow from the winner to the loser and label it with the votes. Do this for the vacation club.

(d) You can also record a tournament's results using a table. Fill this one in:

	... A	... vs H	... vs O
A ...	N/A		
H ...		N/A	
O ...			N/A

- (e) A common way to use the Condorcet tournament is to count how many comparisons each candidate dominates and then pick a winner based on that. Who wins under this system?

- (f) What do you think of this system?

Definitions

- 3. Alice's proposal is called the **Borda count**: everyone ranks all the candidates, and then you assign points, 1 for last, 2 for second-to-last, then 3, and so on. The candidate with the most points wins. This system is common in halls of fame and various other award selections.
 - (a) Can you think of places this is used in real life?

 - (b) Can you think of some pros and cons?

- 4. Nick's proposal is called **Copeland's method**: everyone ranks all the candidates, then you draw a Condorcet tournament to compare them head-to-head and count how many each candidate won (with a tie as half a win).
 - (a) Do you know of places this is used in real life?

 - (b) Can you think of some pros and cons?

Conference locations

A group of mathematicians is selecting a location for an upcoming conference; their options are Seattle, Tacoma, Puyallup, and Olympia. Here is their preference table:

	51	25	10	14
1st choice	S	T	P	O
2nd choice	T	P	T	T
3rd choice	O	O	O	P
4th choice	P	S	S	S

5. Just looking at the table, who do you think should win?

6. Do a Borda count. Who wins?

7. Do Copeland's method. Who wins?

Answers: 1a. A: 19, H: 19, O: 16 1b. A and H tie 2a. yes 2c. H beats A (6:4) and O (6:4); A beats O (7:3) 2d. NA, 4:6; 7:3; 6:4; NA, 6:4; 3:7; 4:6; NA 2e. H wins the most with 2 6. O: 228, P: 194, S: 253, T: 325; T wins 7. O vs P: 65 v 35 (O wins), O v S: 29 v 51 (S wins), O v T: 86 v 14 (T wins), P v S: 49 v 51 (S wins), P v T: 10 v 90 (T wins), S v T 51 v 49 (S wins); S wins

Day 3 - Approval Voting

Vacation destinations, take 3

Look back at the vacation club destination preference schedule one last time. Omar is very disappointed when he realizes Honolulu is a strong contender and says that he can't go if they pick it because the trip will be too long for the amount of vacation time he has saved up at work. He suggests yet another alternative: instead of ranking the cities, maybe they should all mark the ones they can go to and the ones they can't and then see which city is the most accessible for the group as a whole. They do, and they arrive at this table (note: this is not a preference schedule):

	1	1	2	3	2
A	X	X	X		X
H		X		X	X
O	X			X	

1. Which city would win?
2. What do you think of this system?

Definitions

Omar's method is called **approval voting**. Unlike the other systems, the voters don't rank the candidates, they just mark all the ones they find acceptable, and the candidate with the most approvals wins.

3. Do you know of places this is used in real life?
4. Can you think of some pros and cons?

Scholarship committee

A committee is trying to award a scholarship to one of four students, Anna, Brian, Carlos, or Dimitry. Here is their preference schedule (note some committee members decided that some students tied):

	3	2	3	1
1st choice	A	C	B	D
2nd choice	B	A	D	A/B/C
3rd choice	D	B/D	A/C	
4th choice	C			

- Who do you think should win? Examine the committee members' rankings, and think about what a reasonable outcome would be. If you could offer partial scholarships to multiple candidates, would that change your answer?
- What assumptions did you make when making your decision? What factors did you take into account?
- Calculate a winner for the scholarship committee using the plurality system.

8. Use IRV.

9. Use a Borda count.

10. Use Copeland's method.

11. The committee also collects information about which members approve of which candidates. Conduct an approval vote.

	3	2	3	1
A	X	X		
B	X		X	
C		X		
D	X		X	X

12. How do the outcomes compare to the one you picked? Are some of these systems more or less useful for picking a scholarship recipient? Which one most accurately represents the judgment of the committee, and why?

13. The 3 people in the first column of the preference schedule realize that Dimitry will win easily and decide that maybe they don't actually approve of him after all; they did rank him third, and maybe he isn't really that much better than Carlos. They decide to revoke their approval of him and request a revote.

(a) Redraw the approval schedule with their altered vote and conduct a new approval vote. Who wins?

3	2	3	1
A			
B			
C			
D			

(b) These three people are happier with the new results but wonder if they could do even better and get their favorite candidate, Anna, to win. Can they?

(c) Revisit Question 4 and update your answer.

Answers: 1. A: 6, H: 5, O: 9; O wins 7. A: 3, B: 3, C: 2, D: 1; A, B tie 8. D eliminated; 2nd round, A: 3, B: 3, C: 2, C eliminated; 3rd round, A: 5, B: 3, A wins 9 there are several possible ways to adapt a Borda count to accommodate ties; the way Borda originally did it was to give every candidate a point every time they beat another candidate, ex, in the last column, A/B/C get 0 and D gets 3; in this case, A: 13, B: 15, C: 6, D: 12; B wins 10 A v B: 5 v 3 (A wins), A v C: 3 v 2 (A wins), A v D, 5 v 4 (A wins), B v C: 6 v 2 (B wins), B v D: 6 v 1 (B wins), C v D: 2 v 7 (D wins); overall, A wins 3 times, B 2, D 1, C 0, so A wins 11 A: 5, B: 6, C: 2, D: 7; D wins 13a A: 5, B: 6, C: 2, D: 4; B wins 13b they could change their vote so they only approve of A, and then A would win with 5 (unless, of course, the 2 people in the second column start voting insincerely, too)

Day 4 - Fairness

Let's collect some of our ideas about fairness in one place. There are many, but four are commonly discussed:

Majority criterion

Look back at the mathematicians and their vote for a conference location.

1. If the mathematicians just did a plurality vote, what would the results be? Calculate the number of votes and percentage of the total vote each candidate would receive.
2. Who won under plurality, IRV, Borda count, and Copeland's?

This type of winner, someone who gets more than 50% of the first-place choices, is called a **majority winner**. The idea that any majority winner should win is called the **Majority Criterion**.

3. Does the Borda count satisfy the majority criterion?
4. What about plurality, Copeland's, and IRV?

Condorcet criterion

Recall the vacation club vote and the problem Nick raised. He identified a winner who could win head-to-head against any other individual candidate (even though they may or may not win in a full competition). This is called a **Condorcet winner**.

5. Was there a Condorcet winner in the vacation club vote?

6. Who won under the different methods?

The idea that a Condorcet winner should win the election is called the **Condorcet Criterion**.

7. Does Copeland's method satisfy the Condorcet criterion?

8. What about plurality, Borda, and IRV?

Monotonicity Criterion

Three candidates are running for City Council in a small city, Adams, Brown, and Carter.

9. The preference schedule is shown below. Who wins under IRV?

	3700	2200	1200	2900
1st choice	A	B	B	C
2nd choice	B	C	A	A
3rd choice	C	A	C	B

10. After the pollsters collect their data, B is embroiled in a minor scandal, and 1000 of the voters who originally ranked the candidates B, A, C decide to cast their actual votes for A, B, C. Do you think this should affect the outcome?

11. Draw a corrected preference schedule. Who wins under IRV now?

1st choice
2nd choice
3rd choice

We have just developed another criterion for fair elections: that if a candidate manages to steal some voters from someone else, it cannot hurt that candidate. We call this the **Monotonicity Criterion**.

12. Does IRV satisfy the monotonicity criterion?

13. What about plurality? Borda count? Copeland's?

Independence criterion

Peter, John, and Mary are auditioning to be first chair in orchestra. A panel of 8 musicians ranks them:

	4	2	2
1st choice	P	J	M
2nd choice	M	M	J
3rd choice	J	P	P

14. Use plurality. Who wins?

15. Use IRV. Who wins?

16. Use a Borda count. Who wins?

17. Use Copeland's method. Who wins?

18. After the vote, the committee realizes John is only a sophomore, but the first chair must be at least a junior. Should this change the outcome?

19. Rewrite the preference schedule without John. Who should be first chair now?

1st choice

2nd choice

We now have another criterion: the idea that if someone is thrown out, it shouldn't hurt anyone else. This is the **Independence Criterion**.

20. Which systems satisfy the independence criterion?

Summary

21. Fill in the chart below. Which system satisfies which criteria? Make notes for yourself to explain each.

	Condorcet	Monotonicity	Majority	Independence
Plurality				
Copeland's				
IRV				
Borda Count				

Answers: 1. O: 14%, P: 10%, T: 25%, S: 51% 2. Borda: T wins; all others: S wins 3-4. Borda violates it by the example above; Majority satisfies it (think about why - you need to make an argument here, not just refer to one example where it works; you'll discuss this answer in class); IRV and Copeland's do also (hint: think about how the process would go for a majority winner). 5. H 6. plurality/IRV: A wins, Borda: A and H tie, Copeland's: H wins 7-8. plurality, IRV, and Borda violate it by this example; Copeland satisfies it (you'll discuss this answer in class) 9. A wins 11 4700 $A > B > C$, 2200: $B > C > A$, 200: $B > A > C$, 2900: $C > A > B$; C wins 12-13. IRV violates monotonicity by this example; the other systems all satisfy it (hint: think about what could happen if a single voter swapped their opinion of two candidates) 14. P wins 15. P wins 16. M wins 17. M wins 19. 2: $M > P$, 2: $P > M$; in all systems: M, P tie 20. by this example, none of these systems satisfy independence

Examples

3. You are running a plurality election for mayor; candidate A gets 10% of the vote, B gets 44%, and C gets 46%. You declare C to be the winner. Below are possible homework problems and answers, each of which has an error in the answer. Identify the errors.

(a) *Question:* Is the independence Criterion satisfied for this voting system?
Answer: For this election, we have no plans to drop candidates, so it doesn't apply.

(b) *Question:* Is the Majority Criterion satisfied for this voting system?
Answer: No, because no one got a majority.

(c) *Question:* Define the Condorcet Criterion.
Answer: The Condorcet Criterion is a candidate who can beat any other candidate in a head-to-head election.

(d) *Question:* Define the Condorcet Criterion.
Answer: If a candidate wins, then they are a Condorcet winning candidate.

(e) *Question:* Is the Monotonicity Criterion satisfied?
Answer: The Monotonicity Criterion is not relevant because voters are unlikely to change their minds.

Fairness Considerations

4. In the examples above, all the voters were treated exactly the same way, which we call **anonymous** (note this doesn't have anything to do with whether a vote is private, it just has to do with whether the system distinguishes between the voters).
 - (a) Should all the voters be treated the same way?

 - (b) What about citizens in a city selecting the next mayor?

 - (c) What about stockholders voting on a company's board of directors?

 - (d) What about product reviewers for an online retailer?

5. All of the examples above also treat all the candidates exactly the same way, which we call **neutral**.
 - (a) Should all the candidates be treated the same way?

 - (b) What about types of cookies to be served at a dorm party?

 - (c) What about possible destinations for a vacation club?

 - (d) What about candidates for mayor of a city?

 - (e) What about a bill and possible alterations considered by a legislature?

Insincere Voting

6. Consider a plurality election where the preference schedule is shown below. Might voters in the third column be inclined to vote insincerely? Could it affect the outcome?

	96	90	10
1st choice	A	B	C
2nd choice	B	A	B
3rd choice	C	C	A

7. C actually entered the race in Question 6 late. Before they joined, the preference schedule looked like the one below. How might the other voters might perceive candidate C?

	96	100
1st choice	A	B
2nd choice	B	A

8. Consider the election shown below under the Borda method. Explain why voters in the second column might be inclined to vote insincerely. How could it affect the outcome of the election?

	20	18
1st choice	A	B
2nd choice	B	A
3rd choice	C	C

9. Compare the motives of the insincere voters in Questions 6 and 8.