RESEARCH PROFILE

MARGARET I. DOIG

1. Preface

I have prepared this note for my external reviewers to give you a picture of my research program, how it fits into scholarly life at Creighton, and how it impacts and is impacted by the other aspects of my job. My application for promotion and tenure will be reviewed by two committees from across the university; in the words of the College of Arts & Sciences R&T guidelines (para B3), the committees will rely on "peers from appropriate scholarly disciplines ... to evaluate the quality and extent of [candidates'] scholarly achievements according to the standards of the candidates' departments and recognized disciplinary expectations." The standards are summarized in Section 3.

In a forum for R&T applicants, we were encouraged to mention any external factors which may have affected our research output. While at Creighton, I have experienced a number of life events, including the births of my second and third children, the loss of a parent, the loss of two additional pregnancies, and the general disruption of COVID-19. I did not use maternity or FMLA leave beyond two weeks, in part because my department has been short at least one tenure-stream faculty member (out of 9) for most of my time here. I did not apply for pre-tenure sabbatical for the same reason. I have also taught more extensively than I expected; I teach 3-3, and I have specialized in teaching upper division courses, with 15 sections in 14 semesters. This included 9 new preps; I designed the curriculum materials from scratch for the 2 interdisciplinary courses (one with history and one with political science), assembled materials for 2 more from multiple sources, and also created the materials for our core liberal arts math course. Finally, I have participated in service more heavily than expected as well, e.g., serving on 7 hiring committees.

2. Research Program

My research program is eclectic. The research environment at Creighton has resulted in my exposure to a number of fields I had never expected to explore, and my time supervising undergraduate research has turned my attention in new directions. When fully staffed, my department has 9 tenure-stream faculty, including applied math, data science, and statistics. I was not hired to be part of a pre-existing research group, I was hired to be the topology/geometry representative and teach upper division courses and supervise undergraduate research in those areas. As it has worked out, I have supervised more students in discrete math (where I have some background as well), but several of their projects have had a very topological feel.

I have a few remaining topology questions which I work on between other distractions, including a foray into experimental knot theory (Section 2.3) and a question on knot genus. One of my recent projects in graph theory grew out of supervising undergraduate research on an invariant from mathematical chemistry, the Randić index (Section 2.5). I also returned to an old question of interest in graph theory and made some recent progress (Section 2.4). Another project grew out of an interdisciplinary course I taught for our honors program on the mathematics of voting theory, although it was also heavily influenced by a unit we teach in our liberal arts math course, and it has, in turn, heavily influenced the way I rewrote material for that course (Section 2.8). Two more projects grew out of discussions with a colleague who specializes in algebra and fuzzy math: a paper on fuzzy algebra (Section 2.6), and a pair of papers using a measure inspired by fuzzy math to analyze world progress in sustainability (Section 2.7). A final project on matroids (still in the early stages and so not discussed further here) is underway with my applied math colleague and another collaborator.

2.1. **Low-dimensional topology.** My early research was in low-dimensional topology, specifically Heegaard Floer theory. I still have several projects active, but they have taken a back seat to joint projects with current colleagues or which grew out of student research I supervised. I will briefly mention here those projects with preprints or active submissions which have occurred while at Creighton.

Stephan Wehrli and I completed a project [DW] to show combinatorially the homology cobordism classification of lens spaces; specifically, we showed that the spin-c structures and d-invariants of lens spaces were isomorphic in the category of torsors and functions exactly when the lens spaces were oriented homeomorphic, i.e., we showed combinatorially that the d-invariants identify the homeomorphism type of the lens space. The classification of the lens spaces was previously known, but our proof is different because it is combinatorial (modulo the fact that the d-invariants are cobordism invariants). As such, it falls into the old topology practice of translating complex analytical or geometric results into combinatorial terns (see, e.g., [MOS09, MOST07, MOT09]).

This article is currently with a referee at the *NY Journal of Mathematics*; while the project began prior to my arrival at Creighton, shepherding it through the peer-review process has been an on-going activity. When this paper is published, we plan to continue the investigation with homology cobordism classification of surgery on the trefoil, which is not known.

2.2. **Math Toolkit.** I have developed a math toolkit to calculate several invariants in low-dimensional topology and graph theory. It runs on a private server and is available for public use through a user-friendly GUI on my website [Doia].

This started when an early project of mine explored manifolds with finite fundamental group which can be realized as Dehn surgery on a knot in S^3, particularly a hyperbolic knot. The Heegaard Floer correction terms or d-invariants can be used to obstruct such surgeries, and I conducted a computer search to look for exceptional surgeries. The resulting data provided enough intuition to prove the general case [Doi15, Doi16]. I initially wrote very basic c code, but the desire to do larger and larger examples resulted in a series of progressive improvements, including a migration to c++. It is common within low-dimensional topology to make code available to other researchers, but I knew from personal experience that it is not always easy to run or engage with someone else's code, and there are topologists who lack the time or proficiency. In 2019, with the help of an undergraduate assistant who also contributed to the cord code, I set up the public toolkit.

I have added to the toolkit several times since, most notably with a tool to calculate the genus of a knot and whether it is fibered (note: I am not aware of any other available computerized tools to calculate these invariants for an arbitrary knot); a random knot generator and a few additional knot theory invariants behind the project in Section 2.3; and Randić index and graph theory calculations behind the project of Section 2.5. I anticipate adding further capabilities in future.

2.3. **Experimental knot theory.** I had used computer-assisted calculations for years to inform my proof work, but it was not until one of my students proposed some experimental graph theory as a summer project that I considered numerical simulation as an end unto itself. I now have one active project to investigate the distribution of invariants in the grid diagram model of knots [Doid]. Knots as a probability space are an emerging area of topology, e.g., I will joining a Banff workshop in 2024.

Understanding the normative or typical behavior of a knot and the distribution of certain invariants has applications in a number of applied fields where knots (and links) arise naturally, for example, the statistical mechanics of long-chain polymers or the expected topological behavior of DNA. There are some assumptions involved. A knot is typically defined to be an embedding $S^1 \hookrightarrow S^3$, polymers and DNA are often not closed loops, but they are effectively divided into large domains where their ends are functionally fixed away from action, and the topology of these strings appears to influence their behavior in nature, as, for example, the knotting complexity of a polymer can affect its mobility when passing through a resistant environment, as in electrophoresis (e.g., see [SKB+96]). An additional consideration for DNA is that it is two-stranded, so it is perhaps better described by a *framed* knot (a knot with a choice of longitude). Torsional stress on the molecule will often convert any over- or under-twisting into supercoiling of the molecule, or *writhe*, which may may be altered by and inhibit or promote certain genetic processes: picture a pair of twisted headphone cords (the double helix), the ends fixed, and insert your finger between the cords (the RNA polymerase, traveling down the string to replicate it) and pull it from one end to the other, which will result in positive supercoiling ahead and negative supercoiling behind your finger.

I conducted a study of the grid diagram as a knot model and explored writhe with two other invariants, the size of a knot and the number of components. This involved generating random knots and links by two different methods (one guaranteed to produce a single component, the other more representative of what a truly "random" grid diagram could reasonably be). I produced generating functions to verify the size and component count.

Theorem 1. [Doid, Theorem 5.3] *A generating function for* $(1/(n!n!)$ *times) the number of* $n \times n$ *grid diagrams is*

$$g(x) = (1-x)^{-1}e^{-x}$$

and for $(1/(n!n!)$ *times) the number of* $n \times n$ *grid diagrams representing* k*-component links:*

$$G(x,y) = (1-x)^{-y}e^{-xy}.$$

The writhe was significantly more complex, so I completed a thorough numerical simulation and calculated others measures of the distribution.

Observation 2. [Doid, Section 6] *The writhe w of knots in an $n \times n$ grid diagram for $n \leq 100$ follows a distribution with trivial expected value and skewness and with variance and kurtosis given by:*

$$Var(w) = \frac{1}{18}n^2 \qquad \kappa(w) = 3.5.$$

This paper is being resubmitted (please see my website for current status).

2.4. **Grid graphs and run length.** Several of my papers live in or are inspired by discrete math; while not a subfield of topology, the connections in habit and approach between discrete math and low-dimensional topology are certainly well-known and appreciated, and my own experience with graph theory research predates my exposure to topology. One project relates to bounding the maximum run length of a toroidal grid graph [Doib] (note that this paper had a precursor on the arXiv from a 2003 REU project: a referee discovered an oversight in the main proof which I was unable to close at the time; the paper as it stands now is predominantly new work, including the resolution of the issue).

A *toroidal grid graph* is a graph Cartesian product of simple cycles, and a Hamiltonian cycle is a cycle which visits each vertex exactly once. The *run length* of such a cycle in a grid graph is the minimum r so that any r adjacent edges come from different factors of the product. The original example of a grid graph is the set of k-bit binary words, which is the product of k 2-cycles; two such words are adjacent if they differ in exactly one coordinate. A Hamiltonian cycle in this graph is a listing of all the words in such a way that any two adjacent ones differ in only one place, and its run length is the minimum spacing between changing a single bit twice. Such a listing is useful for applications like electronic position-to-digital converters, which use these cycles both to enable error-detection and to minimize movement of a detector head.

The maximum possible run length (or mrl) in a grid graph of k-bit binary words was bounded first [GLN88, GG03], and the grid graphs with maximum run length at least 2 have been classified [RS03]. I studied higher maximum run lengths. In particular, I demonstrated a family of Hamiltonian cycles of nearly ideal run length (one less than the dimension, which is a hard upper bound) for the graphs where all factors were the same size. Additionally, I developed a technique to decompose a grid graph into a product of smaller grid graphs and combine their Hamiltonian cycles in a consistent way to form a cycle in the original graph which is both a simple cycle (as opposed to a union of disjoint cycles) and whose edges are chosen so the run length is at least as large:

Theorem 3. [Doib, Theorem 5.1] *Let G_1 and G_2 be grid graphs with orders n_1 and n_2. If there exist s_1 and s_2 such that $\gcd(n_1, n_2) = s_1 + s_2$ and $\gcd(s_i, n_i) = 1$, then*

$$\operatorname{mrl}(G_1 \times G_2) \geq \min \left\lfloor \frac{s_1 + s_2}{s_i} \operatorname{mrl}(G_i) \right\rfloor.$$

I also developed several explicit theorems applying these techniques to demonstrate several families which have run length at least 3, as well as bound the maximum run length of other families, including:

Theorem 4. [Doib, Theorem 6.1] *Let G be a toroidal grid graph with k terms which share a common prime factor. Then*

$$\operatorname{mrl}(G) \geq k - 1.$$

This project is undergoing final edits before submission. Please see my website for updates.

2.5. **Blocks and mathematical chemistry.** In another project in discrete math, I proved graph radius bounds the Randić index for cactus graphs, in the process studying the relationship between eccentricity and the block structure of a generic graph [Doic]. I stumbled upon this project accidentally after I confidently proclaimed to a colleague it would be easy to find an undergrad research project in mathematical chemistry - and a student showed up in my office a few days later. She ended up working on the Randić index.

The Randić connectivity index is a graph theory invariant originally designed to study the branching of molecules. It is experimentally verified to be associated to boiling and reactivity of hydrocarbons and is now used for predicting biological activity, physicochemical properties, and toxicological responses of chemical compounds based on their molecular structure (see, for example, [KH86, Pog00, GDGdJOP08, TC08, KH76]). If G is a graph showing the structure of a molecule, then define

$$R(G) = \sum_{[u,v]\in E(G)} \frac{1}{\sqrt{\deg(u)\deg(v)}}$$

where $[u, v]$ runs over all the edges of the graph.

The Randić index is worth studying from a graph theoretic point of view, in particular because it identifies a type of branching not easily encapsulated by other invariants. A computer prediction program first identified a possible link to graph radius [Faj88], although it has been resistant to repeated efforts to prove it.

A graph is imbued with a metric which measures the minimum number of edges in a path between vertices, and the *eccentricity* of a vertex is the maximum distance to any other vertex. The maximum eccentricity is the *diameter* of the graph, and the minimum is its *radius* (and a vertex realizing that minimum is a *center*).

A *block* is a maximal subgraph which cannot be disconnected by removing any one vertex, and a *separating vertex* is a vertex whose removal would disconnect the graph. Any graph may be decomposed into distinct blocks which overlap with their neighbors at separating vertices, and this structure is described in something called the *block-cut-tree* or *BC-tree.*

I extended the concepts of eccentricity, radius, center, and related concepts to blocks and studied their relationship to the traditional concepts. In particular, I demonstrated a method to find a subgraph of a given graph with minimal BC-tree but the same radius and center. I refined a famous and very old result that the radius and diameter bound one another, $\operatorname{rad}(G) \leq \operatorname{diam}(G) \leq 2\operatorname{rad}(G)$.

Theorem 5. [Doic, Theorem 4.1] *If there are multiple central blocks in a graph G, then*

$$\operatorname{diam}(G) = 2\operatorname{rad}(G).$$

If there is a unique central block B of diameter $\operatorname{diam}(B)$*, then*

$$2\operatorname{rad}(G) - \operatorname{diam}(B) \leq \operatorname{diam}(G) \leq 2\operatorname{rad}(G)$$

with the lower bound realizable.

As a consequence of these results, I verified the conjectured bound on the Randić index for the family of cactus graphs (whose blocks have a simple structure).

Theorem 6. [Doic, Theorem 5.7] *If G is a cactus graph but not an even path,*

$$R(G) \geq \mathrm{rad}(G).$$

This project is undergoing a rewrite and will be submitted soon. Please see my website for updates.

2.6. **A fuzzy approach to algebra.** My two most recent research projects have involved a foray into the realm of fuzzy math. This field primarily lives in computer science and applied math; it began in the 1960s as a response to early difficulties in natural language processing. Traditionally, set theory allows an item either to belong to a set or not (the membership function takes the values of 0 or 1), for example, a book is either in the set "fewer than 100 pages," or it is not. Fuzzy set theory, on the other hand, allows a concept of partial membership: something may be partially in a set, for example, a book may be 34% in the set of "good books."

More formally, a *fuzzy subset* of a space A is the set A along with a *grade function* $\mu : A \to [0, 1]$ [Zad65]. There are similar fuzzy concepts of groups, homomorphisms, and modules [Pan87, ZA94, ZA95]: a *fuzzy left R-module* (M, μ) is a left R-module M with a function $\mu : M \to [0, 1]$ satisfying a set of conditions making it compatible with the module operations (e.g., $\mu(x + y) \geq \min(\mu(x), \mu(y))$, and $\mu(0) = 1$). Another fuzzy R-module (N, ν) is a *fuzzy R-submodule* of the first if $N \subset M$ and $\nu(x) \leq \mu(x)$ for all $x \in N$.

The set of fuzzy modules and homomorphisms together form a category, and some of the normal category-theoretic concepts like injective modules carry through. D. S. Malik and I extended the concepts of essential extensions and injective hulls to the fuzzy category [DMa]. An *essential extension* of a fuzzy R-module (N, ν) is another fuzzy R-module (M, μ) with the property that every nonzero fuzzy R-submodule of (M, μ) has nontrivial intersection with (N, ν), and an *injective hull* is an injective essential extension. In traditional module theory, it is equivalent for a module to be injective; to be a direct summand of every extension of itself; and to have no proper essential extensions. Additionally, for a submodule N of a module M, it is equivalent for M to be an essential injective extension, a maximal essential extension, or a minimal injective extension; further, given any N, such an M exists. These results carry partially into the category of fuzzy modules:

Theorem 7. [DMa, Theorems 21-22] *Let (M, μ) be a nonzero fuzzy R-module. The following are equivalent:*

- *(M, μ) is injective.*
- *$M = \mathrm{supp}(\mu)$ and (M, μ) is a direct summand of every extension of itself.*
- *(M, μ) has no proper essential extension.*

Theorem 8. [DMa, Theorems 28, 31-32] *If $M = \mathrm{supp}(\mu)$ and (N, ν) is a fuzzy R-submodule of (M, μ), then the following are equivalent:*

- *(M, μ) is an essential injective extension.*
- *(M, μ) is maximal essential extension.*
- *(M, μ) is a minimal injective extension.*

Further, if $N = \mathrm{supp}(\nu)$, then the fuzzy R-submodule (N, ν) has such an injective hull (M, μ).

We also demonstrated examples of $N \neq \mathrm{supp}(\nu)$ without injective hulls, although the conditions of the theorem above may be satisfied even for fuzzy R-modules of smaller support.

This article is currently submitted to *Fuzzy Sets and Systems*.

2.7. **A fuzzy approach to sustainability.** D. S. Malik and I also conducted an analysis of sustainable development using a method ultimately derived from by fuzzy math [DMb, DMc]. In 2015, the UN Member States adopted the 2030 Agenda for Sustainable Development, which includes 17 Sustainable Development Goals. The UN Statistics Division collects data and assesses progress in these Goal areas, and their conclusions are summarized in an annual report from the Secretary-General which is used at a high level to inform international action and policy development.

We employed a measure based on a concept from fuzzy math (the t-norm) to provide an alternate analysis of the data which is very sensitive to changes in the variables over time. Infima and suprema are prevalent in fuzzy math, beginning with the definition of set intersections and unions (e.g., the *intersection* of two fuzzy subsets is the infimum of their grade functions), and Mordeson and Mathew introduced a t-measure to sustainability studies [MM21]. If S is a set of values between 0 and 100, define

$$t(S) = \begin{cases} \max\{s : s \in S\} & \text{if all } s < 50 \\ \min\{s : s \in S\} & \text{if all } s > 50 \\ 50 & \text{else} \end{cases}$$

This measure is particularly suited to a time-series analysis of the UN Sustainable Development Data: a set of variables was identified for each Goal area, and the variables were scored between 0 (the 2.5th percentile) and 100 (fully complying with the 2030 Agenda). The standard analysis consists of averaging each of these variables to give a score for the Goal area, but applying the t-measure instead yields something different and quite interesting. Many of these sets of variables clump for different countries: a country doing well in climate action tends to be (but is not always) doing well in most of the climate action variables, while a country doing poorly in gender equality tends to be doing poorly in most of its variables. The t-measure acknowledges this. If all variables are scored highly for a country, then it selects the lowest; if all are scored poorly, it selects the highest; if, however, a country has mixed high and low scores for variables, the measure returns a placeholder score of 50. If the variables are moving over time as a group from low to high (or vice versa), then this will, in fact, select a leading variable initially, track it upwards to 50, then stall at 50 to indicate a transitional period as the other variables move, then finally track the lagging variable as the entire set climbs above 50. This characteristic makes it especially valuable for time-series analysis as it detects initial or final movement in a group of variables which would be muted by the use of traditional mean.

We include as an example in Figure 1 a graph of Sudan and its development in Goal 9 (Industry, Innovation, and Infrastructure): the score from the t-measure started in the low teens, dropped under 10, and then climbed into the upper 30s. Examining the variables, we see that it initially tracked the leading variable R&D expenditure, which was responsible for the decrease, but it recovered strongly when several variables increased noticeably over this period, led by mobile broadband usage, which the t-measure score actually tracked. A mean would have recorded the climb in this case, although it would been muted slightly by the non-reactive

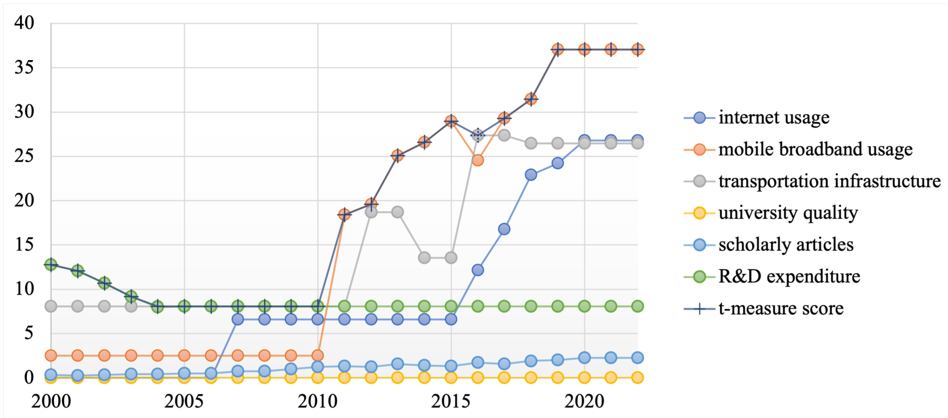

FIGURE 1. Variable scores for Sudan in Goal 9, Industry, Innovation, and Infrastructure, from 2000 to 2022.

variables; this measure is even more valuable in cases where a single, early-warning variable begins to climb before the others move.

When applied to 17 Goal areas for 163 countries, the t-measure detected a large number of trends which were less visible in the traditional analysis and highlighted general worldwide progress (or lack thereof) in sustainable development. We also built an index score for each country based on its Goal scores and examined characteristics of strongly and weakly performing countries and evaluated the impact of income category and geographic region on sustainable development. This body of results is not meant to substitute for the UN's analysis but rather to provide an additional perspective and indicate some patterns which are worth discussing.

This project involved an application of an intriguing and somewhat surprising useful function to analysis of a very significant and well-studied body of data. Several other groups have applied techniques from fuzzy math to data analysis, but none have approached the UN sustainability data on such a wide scope or with the detailed time series analysis we employed. In technique, it is not closely related to my other work, but it borrowed heavily off data analysis skills I learned from teaching them, and I will be incorporating this and related datasets in my classes in future, although perhaps on a more surface level. It can only benefit my students and the world for our community to be learning to understand and interact with large datasets and to apply critical quantitative reasoning to a topic as significant as sustainable development.

These two articles are currently submitted to *New Mathematics and Natural Computation.*

2.8. **Voting theory.** A very surprising recent project was a foray into voting theory, more properly called *social choice theory.* Since before I arrived at Creighton, we have taught a unit on voting theory in our course MTH 205 (Math for the Modern World) taken by humanities majors, nursing students, and some social science students. The mathematical study of elections involves analyzing the precise structure of voting systems, considering how its rules will result in different outcomes based on the makeup of the electorate. This topic is often included in similar liberal arts-style core math courses for two primary reasons: first, an informed citizen should know something about the functioning of electoral systems, and we strive to select topics which are relevant to our students' lives; second, it provides an avenue for teaching logic and logical thinking in a very concrete way which can be more effective for the students. I also developed a new, interdisciplinary mathematics and political science course for our honors program (part of a series entitled "Sources and Methods") consisting of a mathematical analysis of voting theory and an examination of its historical development.

I rewrote the course materials for MTH 205 for use by the whole department, producing a set of interactive worksheets to guide student progress in class. I thought extensively about this section and the ideal way to teach it, especially after teaching some of the same material from a different perspective in the honors course and seeing its historical development. One particularly influential figure was Nicolas de Caritat, marquis de Condorcet, an Enlightenment thinker who was as much philosopher as mathematician. He was not the first mathematician to study voting systems, but he was the first to rigorously review multiple systems for their logical soundness and to consider them explicitly as objects which convert an input (a set of voters' preferences) to an output (a collective decision) which must be reasonably responsive to its input.

I wrote an article [Doi23] to describe how we use voting theory to satisfy our teaching objectives, primarily in teaching logic, and to explore how Condorcet's particular contributions to the field are reflected in our curriculum today. For example, we talk in class about fairness, not in the sense of unrestricted access to electoral mechanisms, but in the sense of determining what kinds of election outcomes are faithful to the individual voters' preferences. We consider many possible sets of input values, some with only slightly varying conditions, and we discuss how the differences can (and ought) to be reflected in the outcome. Early writers had given examples when they described voting systems, but Condorcet was the first to consider the pairing of possible input and output values explicitly as we do in class. He was also the first to do our next activity, namely, we test specific voting systems against particular inputs and check whether the outcomes seem reasonable. Condorcet phrased his analysis of these outcomes using very logical language. For example, he argued that, if a voting system ranks candidate A over B when considered in isolation, and ranks B over C in isolation, then it ought to rank A over C when all three candidates are considered together. Today we call this *transitivity* or *independence*, and we teach it as one of four logical criteria for fairness. This entire exercise quietly teaches a series of important concepts to our students: the difference between a statement and it converse (there is a particularly strong type of candidate called a Condorcet winner, and one of the criteria says, "A Condorcet winner will win the election," which is not the same as, "The election winner will be a Condorcet winner"); the difference between "there exists" and "for all" ("A Condorcet winner will win" vs "The Condorcet winner will win"); even the difference between "proof by example" and "proof by counterexample" ("Does the plurality system satisfy the Condorcet Criterion? No? OK, is it enough just to give an example?").

This article is scheduled to appear this summer in *XVIII New Perspectives on the Eighteenth Century*. A second research idea stemmed from the voting theory course which I hope to have time to pursue in future.

3. Standards

These are the Department of Mathematics and Division of Science guidelines for evaluating scholarship for tenure and promotion to associate professor which the two R&T committees will evaluate for me with your help.

A trajectory of scholarship, including 3-4 peer-reviewed publications since coming to Creighton. The actual number expected will depend on factors such as the importance and extent of the work and other professional demands placed on faculty member's time.

I have 5 peer-reviewed publications, 1 of which has appeared since coming to Creighton, and 3 more currently undergoing the peer-review process (although that number should soon increase). I will leave it to others to judge their importance, but I believe that each of my articles contributes something to the academic or wider world.

I also have 3 non-peer-reviewed products, a toolkit of implemented topology operations; an editorial on statistic-based risk management; and a set of curriculum materials for a liberal arts math class. While not peer-reviewed publications, they constitute a meaningful part of my research life and a way that I am active in the world as a scholar.

Please refer to my website for current status and copies of all publications, including preprints.

Note to the reviewers: There is no restriction on the subject matter of publications, merely that they be "peer-reviewed," "scholarly," and "relevant to the discipline"; in particular, they may be interdisciplinary, and acceptable topics include "scholarship of the discipline," as the article on Condorcet is.

A record of presentations at regional and national professional meetings, or external department colloquia, typically averaging 1 per year.

Since coming to Creighton, I have averaged 1.1 external department colloquia or presentations at regional/national meetings per year, in addition to a few more expository external presentations; this was lower than my pre-Creighton average of 2.2/year because of COVID-19 and having two more children.

Note to the reviewers: It is unclear from the guidelines whether department colloquia count as research presentations the way that conference talks do. The R&T committees may appreciate any insight you can provide into the norms for the field.

Submission of one or more grant applications, including one or more extramural applications since coming to Creighton.

I received a Summer Faculty Research Fellowship from Creighton University, which funded me and an undergraduate assistant for the summer. While a postdoc, I received an NSF-AWM travel grant and applied for an NSF research grant. I plan to apply for an additional travel or similar grant this fall.

Note to the reviewers: The R&T committees have indicated that they will look to the external reviewers to help them determine what variety of grants would be normal for someone like me (13 years out of grad school with a 3-3 teaching load and no grad students), whether a traditional NSF grant or something more similar to the NSF-AWM travel grant.

Additional factors, such as successful inclusion of undergraduate students into research; record of service to the discipline, including as a peer-reviewer or conference organizer.

I have supervised 6 Creighton undergraduates performing research and assisted with 1 more; 2 have presented research posters, 1 contributed code to my math toolkit, and 2 are preparing articles for submission.

Since coming to Creighton, I have served as a peer-reviewer twice and as a textbook reviewer once and helped organize a conference special session once.

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E-mail address: margaretdoig@creighton.edu

Department of Mathematics, College of Arts and Sciences, Creighton University